**Studying the MS-optimal design**

The design parameter consists of 2 treatments, 3 biological replicates, i.e. 6 animals, and 2 technical replicates in the first phase experiment. The second phase experiment uses 3 runs and 4 tags.

This write-up investigates 1 of 54 MS-optimal design that was founded using the two-stage process. The first stage is to find the top 250 designs based on maximising the trace of the information matrix

The following design was founded.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Phase 1 experiment** | | | **Phase 2 experiment** | |
| **Ani** | **Trt** | **TechRep** | **Run** | **Tag** |
| A | a | 2 | 1 | 1 |
| B | b | 1 | 1 | 2 |
| F | b | 2 | 1 | 3 |
| E | a | 1 | 1 | 4 |
| B | b | 2 | 2 | 1 |
| D | b | 1 | 2 | 2 |
| E | a | 2 | 2 | 3 |
| C | a | 1 | 2 | 4 |
| F | b | 1 | 3 | 1 |
| C | a | 2 | 3 | 2 |
| D | b | 2 | 3 | 3 |
| A | a | 1 | 3 | 4 |

The allocation of animals to runs and tags can be presented in the following table as

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Run | Tag | | | |
| **1** | **2** | **3** | **4** |
| 1 | A | B | F | E |
| 2 | B | D | E | C |
| 3 | F | C | D | A |

The allocation of animals to runs and tags is binary where each run does not occur more than once in every run and tag. This binary design can be confirmed from the animal incidence matrix with respect to runs and tags where the matrices’ elements only contain zero or one.

The animal incidence matrices with respect to runs and tags can be presented as follows

[,1] [,2] [,3]

[1,] 1 0 1

[2,] 1 1 0

[3,] 0 1 1

[4,] 0 1 1

[5,] 1 1 0

[6,] 1 0 1

[,1] [,2] [,3] [,4]

[1,] 1 0 0 1

[2,] 1 1 0 0

[3,] 0 1 0 1

[4,] 0 1 1 0

[5,] 0 0 1 1

[6,] 1 0 1 0

Allocation of treatments to the runs and tags is shown as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Run | Tag | | | |
| **1** | **2** | **3** | **4** |
| 1 | a | b | b | a |
| 2 | b | b | a | a |
| 3 | b | a | b | a |

|  |
| --- |
| **Information matrix associated with the animal effects in the Between Runs stratum**  [,1] [,2] [,3] [,4] [,5] [,6]  [1,] 0.16666667 -0.08333333 -0.08333333 -0.08333333 -0.08333333 0.16666667  [2,] -0.08333333 0.16666667 -0.08333333 -0.08333333 0.16666667 -0.08333333  [3,] -0.08333333 -0.08333333 0.16666667 0.16666667 -0.08333333 -0.08333333  [4,] -0.08333333 -0.08333333 0.16666667 0.16666667 -0.08333333 -0.08333333  [5,] -0.08333333 0.16666667 -0.08333333 -0.08333333 0.16666667 -0.08333333  [6,] 0.16666667 -0.08333333 -0.08333333 -0.08333333 -0.08333333 0.16666667    [,1] [,2] [,3] [,4] [,5] [,6]  [1,] 1/6 -1/12 -1/12 -1/12 -1/12 1/6  [2,] -1/12 1/6 -1/12 -1/12 1/6 -1/12  [3,] -1/12 -1/12 1/6 1/6 -1/12 -1/12  [4,] -1/12 -1/12 1/6 1/6 -1/12 -1/12  [5,] -1/12 1/6 -1/12 -1/12 1/6 -1/12  [6,] 1/6 -1/12 -1/12 -1/12 -1/12 1/6  **Canonical efficiency factors associated with the animal effects in the Between Runs stratum** |
| [1] 0.25 0.25 |
|  |
|  |
| **Average efficiency factor associated with the animal effects in the Between Runs stratum** |
| [1] 0.25 |
| **Information matrix associated with the animal effects in the Within Runs stratum**  [,1] [,2] [,3] [,4] [,5] [,6]  [1,] 1.50 -0.25 -0.25 -0.25 -0.25 -0.50  [2,] -0.25 1.50 -0.25 -0.25 -0.50 -0.25  [3,] -0.25 -0.25 1.50 -0.50 -0.25 -0.25  [4,] -0.25 -0.25 -0.50 1.50 -0.25 -0.25  [5,] -0.25 -0.50 -0.25 -0.25 1.50 -0.25  [6,] -0.50 -0.25 -0.25 -0.25 -0.25 1.50 |
| **Canonical efficiency factors associated with the animal effects in the Within Runs stratum** |
| [1] 1.00 1.00 1.00 0.75 0.75 |
|  |
| **Average efficiency factor associated with the animal effects in the Within Runs stratum** |
| [1] 0.8823529 |

**Theoretical ANOVA of this design can be expressed as follows,**

|  |
| --- |
| $ANOVA |
| DF e Ani Run |
| Between Run |
| Between Ani 2 1 1/2 4 |
| Within |
| Between Ani |
| Tag 3 1 11/6 0 |
| Residual 2 1 7/4 0 |
| Residual |
| Tag 3 1 0 0 |
| Residual 1 1 0 0 |
|  |
| $EF |
| Tag Trt eff.Tag eff.Trt |
| Between Run |
| Between Ani |
| Within |
| Between Ani |
| Tag 12/11 6 4/11 1 |
| Residual |
| Tag 15/8 5/8 |

Based on this ANOVA table, the formal test for the treatment effects cannot be done, because the tag effects are completely confounded with the treatment effects. The fixed part of the ANOVA table shows that all the treatment information is in the Tag MS of the Between Animals Within Runs stratum. However, the amount of tag information presented in Between Animals (4/11) and Within Animals (5/8) does not add up to one. This means the separation of the tag information of all three tag contrast is not even between the Between Animals and Within Animals strata. In addition, two of the five DF associate with the animal effects is partially confounded with Between Run stratum with 0.25 of the animal information; hence, there is still five DF associated with animal effects in the Within Runs stratum.

The first step is to fit five animal contrasts to study how the animal information is split across the Between Runs and Within Runs strata.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  | | --- | | **Basic animal contrasts in the Within Runs stratum derived from the eigenvectors of animal effects in the Within Runs stratum** | | [,1] [,2] [,3] [,4] [,5] [,6] | | [1,] 7.071068e-01 0.000000e+00 0.000000e+00 0.5773503 0.000000e+00 -0.4082483 | | [2,] -2.220446e-16 -6.654677e-01 -2.390663e-01 -0.2886751 -5.000000e-01 -0.4082483 | | [3,] 8.326673e-17 -2.390663e-01 6.654677e-01 -0.2886751 5.000000e-01 -0.4082483 | | [4,] -3.608225e-16 2.390663e-01 -6.654677e-01 -0.2886751 5.000000e-01 -0.4082483 | | [5,] 8.326673e-17 6.654677e-01 2.390663e-01 -0.2886751 -5.000000e-01 -0.4082483 | | [6,] -7.071068e-01 1.110223e-16 5.134781e-16 0.5773503 -5.551115e-17 -0.4082483 |   **Theoretical ANOVA of design 1 after fitting the animal contrasts, i.e. Basic animal contrasts in the Within Runs stratum** | |
| $ANOVA | |
| DF e Ani Run | |
| Between Run | |
| Between Ani.blk4 1 1 1/2 4 | |
| Between Ani.blk5 1 1 1/2 4 | |
| Within | |
| Between Ani.blk1 | |
| Tag 1 1 2 0 | |
| Between Ani.blk2 | |
| Tag 1 1 2 0 | |
| Between Ani.blk3 | |
| Tag 1 1 2 0 | |
| Between Ani.blk4 | |
| Tag 1 1 3/2 0 | |
| Between Ani.blk5 | |
| Tag 1 1 3/2 0 | |
| Residual | |
| Tag 3 1 0 0 | |
| Residual 1 1 0 0 | |
|  | |
| $EF | |
| Tag Trt eff.Tag eff.Trt | |
| Between Run | |
| Between Ani.blk4 | |
| Between Ani.blk5 | |
| Within | |
| Between Ani.blk1 | |
| Tag 1/2 2 1/6 1/3 | |
| Between Ani.blk2 | |
| Tag 2253340/2389931 8/11 194395/618536 4/33 | |
| Between Ani.blk3 | |
| Tag 6473/11618 36/11 2429/13079 6/11 | |
| Between Ani.blk4 | |
| Tag 1 1/3 | |
| Between Ani.blk5 | |
| Tag 1/3 1/9 | |
| Residual | |
| Tag 15/8 5/8 | |
| This theoretical ANOVA table does not look very clean, because there is always some tag information in every animal contrast specified. | |
| Since this MS-optimal design is founded from attempting to maximise the animal information in the Within Runs and Tags stratum, the animal contrast should be obtain from the eigenvectors of information matrix associated with the animals effects in the Within Runs and Tags stratum. | |
| **Basic animal contrasts in the Within Runs and Tags stratum derived from the eigenvectors of animal effects in the Within Runs and Tags stratum** | |
| [,1] [,2] [,3] [,4] [,5] [,6] | |
| [1,] 5.000000e-01 0.2886751 5.000000e-01 0.000000e+00 5.000000e-01 -0.4082483 | |
| [2,] -2.220446e-16 -0.5773503 -2.220446e-16 -7.071068e-01 -5.828671e-16 -0.4082483 | |
| [3,] -5.000000e-01 0.2886751 5.000000e-01 -1.387779e-16 -5.000000e-01 -0.4082483 | |
| [4,] 5.000000e-01 0.2886751 -5.000000e-01 -1.804112e-16 -5.000000e-01 -0.4082483 | |
| [5,] -3.608225e-16 -0.5773503 -3.191891e-16 7.071068e-01 -6.245005e-16 -0.4082483 | |
| [6,] -5.000000e-01 0.2886751 -5.000000e-01 -4.996004e-16 5.000000e-01 -0.4082483 | |
|  | |
| [,1] [,2] [,3] [,4] [,5] [,6] | |
| [1,] 1/2 75658/262087 1/2 0 1/2 -8721/21362 | |
| [2,] 0 -571/989 0 -5741/8119 0 -8721/21362 | |
| [3,] -1/2 75658/262087 1/2 0 -1/2 -8721/21362 | |
| [4,] 1/2 75658/262087 -1/2 0 -1/2 -8721/21362 | |
| [5,] 0 -571/989 0 2378/3363 0 -8721/21362 | |
| [6,] -1/2 75658/262087 -1/2 0 1/2 -8721/21362 | |
|  | |
| **Canonical efficiency factors associated with the animal effects eliminating the tag effects in the Within Runs stratum** | |
| [1] 1.0000000 0.7500000 0.6666667 0.6666667 0.4166667 | |
| **Information matrix associated with the animal effects eliminating the tag effects in the Within Runs stratum**  [,1] [,2] [,3] [,4] [,5] [,6]  [1,] 1.16666667 -0.2500000 -0.25000000 0.08333333 -0.2500000 -0.50000000  [2,] -0.25000000 1.1666667 -0.25000000 -0.25000000 -0.1666667 -0.25000000  [3,] -0.25000000 -0.2500000 1.16666667 -0.50000000 -0.2500000 0.08333333  [4,] 0.08333333 -0.2500000 -0.50000000 1.16666667 -0.2500000 -0.25000000  [5,] -0.25000000 -0.1666667 -0.25000000 -0.25000000 1.1666667 -0.25000000  [6,] -0.50000000 -0.2500000 0.08333333 -0.25000000 -0.2500000 1.16666667  [,1] [,2] [,3] [,4] [,5] [,6]  [1,] 7/6 -1/4 -1/4 1/12 -1/4 -1/2  [2,] -1/4 7/6 -1/4 -1/4 -1/6 -1/4  [3,] -1/4 -1/4 7/6 -1/2 -1/4 1/12  [4,] 1/12 -1/4 -1/2 7/6 -1/4 -1/4  [5,] -1/4 -1/6 -1/4 -1/4 7/6 -1/4  [6,] -1/2 -1/4 1/12 -1/4 -1/4 7/6  The initial step is to fit the runs as block effects and animal contrasts defined above as the fixed effects. This can help us to see how the animals are decomposed in the Between Runs and Within Runs strata. | |
| |  | | --- | | **Theoretical ANOVA of design 1 after fitting the animal contrasts, treating the animals as the fixed effects** | | $ANOVA | | DF e Run | | Between Run | | Ani.blk2 1 1 4 | | Ani.blk5 1 1 4 | | Within | | Ani.blk1 1 1 0 | | Ani.blk2 1 1 0 | | Ani.blk3 1 1 0 | | Ani.blk4 1 1 0 | | Ani.blk5 1 1 0 | | Residual 4 1 0 | |  | | $EF | | | Ani.blk1 Ani.blk2 Ani.blk3 Ani.blk4 Ani.blk5 eff.Ani.blk1 eff.Ani.blk2 eff.Ani.blk3 eff.Ani.blk4 eff.Ani.blk5 | | | Between Run | | | Ani.blk2 1/2 1/4 | | | Ani.blk5 1/2 1/4 | | | Within | | | Ani.blk1 2 1 | | | Ani.blk2 3/2 3/4 | | | Ani.blk3 2 1 | | | Ani.blk4 2 1 | | | Ani.blk5 3/2 3/4 | | | |
| This theoretical ANOVA table shows two of five animal contrasts, animal contrasts 2 and 5, are partially confounded with Between Runs stratum by 1/4 of animal information. The remaining 3 animal contrasts are orthogonal to the Between Runs stratum, which means 100% of all the animal information for these 3 animal contrasts is intact in the Within Runs stratum. | |
| The next step is to fit the tag contrasts into the ANOVA table. The contrasts are also constructed by the eigenvector. There are two set of contrasts can be generated, the first set of contrast associated with tag effects in the Between Animals Within Runs stratum and tag effects in the Within Animals Within Runs stratum. | |
| **Basic tag contrasts from Between Animals Within Runs stratum** | |
| [,1] [,2] [,3] [,4] | |
| [1,] 7.071068e-01 0.5 0.000000e+00 -0.5 | |
| [2,] -7.071068e-01 0.5 -9.294611e-16 -0.5 | |
| [3,] 2.220446e-16 -0.5 -7.071068e-01 -0.5 | |
| [4,] -6.661338e-16 -0.5 7.071068e-01 -0.5 | |
|  | |
|  | |
| [,1] [,2] [,3] [,4] | |
| [1,] 2378/3363 1/2 0 -1/2 | |
| [2,] -5741/8119 1/2 0 -1/2 | |
| [3,] 0 -1/2 -5741/8119 -1/2 | |
| [4,] 0 -1/2 2378/3363 -1/2 | |
| |  | | --- | | **Information matrix associated with the tag effects in Between Animals Within Runs stratum** |   [,1] [,2] [,3] [,4]  [1,] 0.9166667 -0.4166667 -0.25 -0.25  [2,] -0.4166667 0.9166667 -0.25 -0.25  [3,] -0.2500000 -0.2500000 0.75 -0.25  [4,] -0.2500000 -0.2500000 -0.25 0.75    [,1] [,2] [,3] [,4]  [1,] 11/12 -5/12 -1/4 -1/4  [2,] -5/12 11/12 -1/4 -1/4  [3,] -1/4 -1/4 3/4 -1/4  [4,] -1/4 -1/4 -1/4 3/4 | |
| **Canonical efficiency factors associated with the tag effects in Between Animals Within Runs stratum** | |
| [1] 0.4444444 0.3333333 0.3333333 | |
|  | |
| **Average efficiency factor associated with the tag effects in Between Animals Within Runs stratum** | |
| [1] 0.3636364 | |
| **Basic tag contrasts from Within Animals Within Runs stratum** |
| [,1] [,2] [,3] [,4] |
| [1,] 0.000000e+00 0.5 7.071068e-01 -0.5 |
| [2,] -7.126307e-16 0.5 -7.071068e-01 -0.5 |
| [3,] -7.071068e-01 -0.5 4.440892e-16 -0.5 |
| [4,] 7.071068e-01 -0.5 -3.885781e-16 -0.5 |
|  |
|  |
| [,1] [,2] [,3] [,4] |
| [1,] 0 1/2 2378/3363 -1/2 |
| [2,] 0 1/2 -5741/8119 -1/2 |
| [3,] -5741/8119 -1/2 0 -1/2 |
| [4,] 2378/3363 -1/2 0 -1/2 |
|  |
| **Information matrix associated with the tag effects in Within Animals Within Runs stratum**  [,1] [,2] [,3] [,4]  [1,] 1.3333333 -0.3333333 -0.5 -0.5  [2,] -0.3333333 1.3333333 -0.5 -0.5  [3,] -0.5000000 -0.5000000 1.5 -0.5  [4,] -0.5000000 -0.5000000 -0.5 1.5  [,1] [,2] [,3] [,4]  [1,] 4/3 -1/3 -1/2 -1/2  [2,] -1/3 4/3 -1/2 -1/2  [3,] -1/2 -1/2 3/2 -1/2  [4,] -1/2 -1/2 -1/2 3/2 |
| **Canonical efficiency factors associated with the tag effects in Between Animals Within Runs stratum** |
| [1] 0.6666667 0.6666667 0.5555556 |
|  |
|  |
| **Average efficiency factor associated with the tag effects in Between Animals Within Runs stratum** |
| [1] 0.625  Theoretical ANOVA table of a single phase experiment with runs are fitted as block effects and tags are fitted as fixed effects can be shown as below. Three tag contrasts that are defined from the basic tag contrasts in the Between Animals Within Runs stratum.   |  | | --- | | $ANOVA | | DF e Run | | Between Run 2 1 4 | | Within | | Tag.1 1 1 0 | | Tag.2 1 1 0 | | Tag.3 1 1 0 | | Trt 1 1 0 | | Residual 5 1 0 | | $EF | | Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt | | Between Run | | Within | | Tag.1 3 1 | | Tag.2 3 2/3 1 1/9 | | Tag.3 3 4/3 1 2/9 | | Trt 4 2/3 | | This theoretical ANOVA table shows the tags effects are orthogonal to the Runs the tag information for each of three Tag contrasts are in the Within Runs stratum.  The next theoretical ANOVA table of a single phase experiment with animals are fitted as block effects and tags are fitted as fixed effects can be shown as below. | | $ANOVA | | DF e Ani | | Between Ani | | Tag.1 1 1 2 | | Tag.2 1 1 2 | | Tag.3 1 1 2 | | Residual 2 1 2 | | Within | | Tag.1 1 1 0 | | Tag.2 1 1 0 | | Tag.3 1 1 0 | | Residual 3 1 0 | |  | | $EF | | Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt | | Between Ani | | Tag.1 1 1/3 | | Tag.2 1 2 1/3 1/3 | | Tag.3 1 4 1/3 2/3 | | Within | | Tag.1 2 2/3 | | Tag.2 2 2/3 | | Tag.3 2 2/3 | |  | | The ANOVA table shows all three tag contrasts are separated even with 1/3 and 2/3 into Between Animals and Within Animals strata, respectively.  The next ANOVA table fits Run as the block effects of the Phase 2 experiment, Animal as the block effect of the Phase 1 experiment and the same 3 Tag contrasts which were fitted before. | | **Theoretical ANOVA of design 1 after fitting the first set of contrasts, i.e. Basic tag contrasts from Between Animals Within Runs stratum** | | $ANOVA | | DF e Ani Run | | Between Run | | Between Ani 2 1 1/2 4 | | Within | | Between Ani | | Tag.1 1 1 3/2 0 | | Tag.2 1 1 2 0 | | Tag.3 1 1 2 0 | | Residual 2 1 7/4 0 | | Residual | | Tag.1 1 1 0 0 | | Tag.2 1 1 0 0 | | Tag.3 1 1 0 0 | | Residual 1 1 0 0 | | $EF | | Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt | | Between Run | | Between Ani | | Within | | Between Ani | | Tag.1 4/3 4/9 | | Tag.2 1 2 1/3 1/3 | | Tag.3 1 4 1/3 2/3 | | Residual | | Tag.1 5/3 5/9 | | Tag.2 2 2/3 | | Tag.3 2 2/3 | |  | | After fitting the Run component into this ANOVA, only two of three tag contrasts are separated with 1/3 and 2/3 into Between Animals and Within Animals strata, respectively. However, the first tag contrast is separated with 4/9 and 5/9 into Between Animals and Within Animals strata, respectively. This means the amount of the tag information for the tag contrast one has increase by 1/9. This tag contrast is comparing between the Tag 1 and 2.  The final ANOVA table includes the five Animal contrasts that defined from the eigenvectors. This can help us to sturdy the relationship between the contrasts associated with the animal and tag effects and to see which Animal contrasts are confounded with the Tag contrasts. | | **Theoretical ANOVA of design 1 after fitting the both contrasts i.e. Basic tag contrasts from Between Animals Within Runs stratum and Basic animal contrasts in the Within Runs and Tags stratum** | | $ANOVA | | DF e Ani Run | | Between Run | | Between Ani.blk2 1 1 1/2 4 | | Between Ani.blk5 1 1 1/2 4 | | Within | | Between Ani.blk1 1 1 2 0 | | Between Ani.blk2 1 1 3/2 0 | | Between Ani.blk3 | | Tag.3 1 1 2 0 | | Between Ani.blk4 | | Tag.2 1 1 2 0 | | Between Ani.blk5 | | Tag.1 1 1 3/2 0 | | Residual | | Tag.1 1 1 0 0 | | Tag.2 1 1 0 0 | | Tag.3 1 1 0 0 | | Residual 1 1 0 0 | | $EF | | Tag.1 Tag.2 Tag.3 Trt eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Trt | | Between Run | | Between Ani.blk2 | | Between Ani.blk5 | | Within | | Between Ani.blk1 | | Between Ani.blk2 | | Between Ani.blk3 | | Tag.3 1 4 1/3 2/3 | | Between Ani.blk4 | | Tag.2 1 2 1/3 1/3 | | Between Ani.blk5 | | Tag.1 4/3 4/9 | | Residual | | Tag.1 5/3 5/9 | | Tag.2 2 2/3 | | Tag.3 2 2/3 | |

By fitting both the animal contrasts with the tag contrasts into the ANOVA table, the table shows that the Animal contrasts 3, 4 and 5 are confounded with the Tag contrasts 3, 2 and 1, respectively. Since the Animal contrast 5 also confounded with runs by 1/4 of animals information, this may be the reason why the Tag 1 contrasts has higher amount of tag information compare to the other two tag contrasts. The Animal contrast 5 is comparing Animal A & F versus C & D.

In summary, that amount of the animal information in the Within Runs stratum for this contrast is 3/4 and amount of tag information in the Between Animals stratum without fitting the runs is 1/3. After the Runs are fitted, amount of Tag information in the Between Animals Within Runs stratum on one specific set of tag and animal contrast becomes 4/9, i.e. Tag contrasts 1 and Animal contrast 5. The tag contrast 1 is comparing between the Tag 1 and 2 and the Animal contrast 5 is comparing Animal A & F versus C & D.

From these strange proportions of the efficiency factors, it reminds me of that the “order of fitting” can affect the structure of the ANOVA table. I have generated two ANOVA tables of single phase experiments with different fitting orders of animals and tags.

For the following ANOVA table, the runs are considered as block effects and animals and treatments are considered as fixed effects. I have used the same animal and tag contrasts derived from the eigenvectors. With five animal contrasts are fitted before the three tags contrasts, the following theoretical ANOVA table can be expressed as

$ANOVA

DF e Run

Between Run

Ani.blk5 1 1 4

Ani.blk2 1 1 4

Within

Ani.blk5 1 1 0

Ani.blk1 1 1 0

Ani.blk2 1 1 0

Ani.blk3 1 1 0

Ani.blk4 1 1 0

Tag.1 1 1 0

Tag.2 1 1 0

Tag.3 1 1 0

Residual 1 1 0

$EF

Ani.blk5 Ani.blk1 Ani.blk2 Ani.blk3 Ani.blk4 Tag.1 Tag.2 Tag.3 eff.Ani.blk5 eff.Ani.blk1 eff.Ani.blk2 eff.Ani.blk3 eff.Ani.blk4 eff.Tag.1 eff.Tag.2 eff.Tag.3

Between Run

Ani.blk5 1/2 1/4

Ani.blk2 1/2 1/4

Within

Ani.blk5 3/2 4/3 3/4 4/9

Ani.blk1 2 1

Ani.blk2 3/2 3/4

Ani.blk3 2 1 1 1/3

Ani.blk4 2 1 1 1/3

Tag.1 5/3 5/9

Tag.2 2 2/3

Tag.3 2 2/3

Note that the Animal contrasts 5 and 2 are the two of five animal contrasts that are confounded with the runs by 1/4 of the animal information. The remaining animal MS associated with Animal contrasts of 1, 3 and 4 have all animal information in the Within Runs stratum. In addition, the Tag contrasts 1, 2 and 3 are shown to be confounded with Animal contrasts 5, 4 and 3, respectively, by 4/9, 1/3 and 1/3 of the tag information. Hence, the remaining tag information for Tag contrasts 1, 2 and 3 are 5/9, 2/3 and 2/3, respectively. These efficiency factors the same as the last theoretical ANOVA table when the experiment is treated as two-phase.

The next step is to fit the three tag contrasts before the five animal contrasts; the following theoretical ANOVA table can be expressed as

$ANOVA

DF e Run

Between Run

Ani.blk5 1 1 4

Ani.blk2 1 1 4

Within

Tag.1 1 1 0

Tag.2 1 1 0

Tag.3 1 1 0

Ani.blk5 1 1 0

Ani.blk1 1 1 0

Ani.blk2 1 1 0

Ani.blk3 1 1 0

Ani.blk4 1 1 0

Residual 1 1 0

$EF

Tag.1 Tag.2 Tag.3 Ani.blk5 Ani.blk1 Ani.blk2 Ani.blk3 Ani.blk4 eff.Tag.1 eff.Tag.2 eff.Tag.3 eff.Ani.blk5 eff.Ani.blk1 eff.Ani.blk2 eff.Ani.blk3 eff.Ani.blk4

Between Run

Ani.blk5 1/2 1/4

Ani.blk2 1/2 1/4

Within

Tag.1 3 2/3 1 1/3

Tag.2 3 2/3 1 1/3

Tag.3 3 2/3 1 1/3

Ani.blk5 5/6 5/12

Ani.blk1 2 1

Ani.blk2 3/2 3/4

Ani.blk3 4/3 2/3

Ani.blk4 4/3 2/3

Note that 100% of tag information is now present in tag MS associated with each of these three tag contrasts. The animal contrasts 2 and 5 still confounded with runs by 1/4 of the animal information. The tag MS associated with tag contrasts 1, 2 and 3 all have 1/3 of the animal information. Hence, the remaining animal information, based on subtraction, for Animal contrasts 1, 2, 3, 4 and 5 are 1, 3/4, 2/3, 2/3 and 5/12, respectively. The amount of the animal information for animal contrast 5 in the Within Runs stratum is 1/3 and 5/12 for Tag contrast 1 and Animal contrasts 5, respectively. Record from the previous theoretical ANOVA table, the amount of the tag information associated with tag contrast 1 in the Within Runs stratum is 4/9 and 5/9 for Animal contrasts 5 and Tag contrast 1, respectively. The ratio of 1/3 to 5/12 works out to be identical to the ratio of 4/9 to 5/9.